Effects of the continuum coupling on spin-orbit splitting

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Abstract. Recently, the shell model in the complex k-plane (the so-called Gamow Shell Model) has been formulated using a complex Berggren ensemble representing bound single-particle states, single-particle resonances, and non-resonant continuum states. The single-particle basis used is that of the Hartree-Fock potential generated self-consistently by a finite-range residual interaction. In this framework, we shall discuss the "spin-orbit splitting" of the $3/2₁⁻$ and $1/2₁⁻$ states in ⁷He. It is demonstrated that the continuum effects are very important and cannot be taken into account in standard shell-model calculations.

PACS. 21.60.Cs Shell model – 24.10.Cn Many-body theory – 27.20.+n $6 \le A \le 19$

1 Introduction

It is extremely difficult to describe weakly bound states or resonances in a closed-system formalism such as the nuclear Shell Model (SM). The binding of nuclei close to the particle drip lines depends sensitively both on the coupling to the scattering continuum and on the detailed features of an effective NN interaction [\[1\]](#page-1-0); hence it has to be described in the open-system formalism allowing for configuration mixing, such as the continuum shell model (see ref. [\[2\]](#page-1-1) for a recent review) and, most recently, the Gamow Shell Model (GSM) [\[3,](#page-1-2)[4,](#page-1-3)[5,](#page-1-4)[6\]](#page-1-5) (see also refs. [\[7,](#page-1-6) [8,](#page-1-7)[9\]](#page-1-8)). GSM is the multi-configurational shell model with a single-particle (s.p.) basis given by the Berggren ensemble [\[10,](#page-1-9)[11,](#page-1-10)[12\]](#page-1-11) which consists of Gamow (or resonant) states and the complex non-resonant continuum. The s.p. Berggren basis is generated by a finite-depth potential, and the many-body states are obtained in shell-model calculations as the linear combination of Slater determinants spanned by resonant and non-resonant s.p. basis states. Hence, both continuum effects and correlations between nucleons are taken into account simultaneously. All details of the formalism can be found in ref. [\[4\]](#page-1-3), in which the GSM was applied to many-neutron configurations in neutron-rich helium and oxygen isotopes.

Even though the effective interaction theory for open quantum many-body systems has not yet been developed, recent investigations [\[13,](#page-1-12)[14\]](#page-1-13) in the framework of the Shell Model Embedded in the Continuum (SMEC) [\[15,](#page-1-14)[2\]](#page-1-1) determined basic features of the correction to the eigenenergy of the closed quantum system due to the continuum coupling. The novel feature, absent in the standard SM, is a strong influence of the poles of the scattering (S) matrix on the weakly bound/unbound states. In particular, for nucleons in low- ℓ orbits $(\ell = 0, 1)$, the coupling is singular at the particle emission threshold if the pole of the Smatrix lies at the threshold $[13,14]$ $[13,14]$. Such a coupling may induce the non-perturbative rearrangement of the wave function. Below, we shall illustrate this effect in the case of the spin-orbit splitting in ⁷He.

2 Description of the calculation

In our previous studies $[3,4]$ $[3,4]$, we have used the s.p. basis generated by a Woods-Saxon (WS) potential which was adjusted to reproduce the s.p. energies in 5 He. This potential ("⁵He" parameter set $[4]$) is characterized by the radius $R = 2$ fm, the diffuseness $d = 0.65$ fm, the strength of the central field $V_0 = 47$ MeV, and the spin-orbit strength $V_{\rm so} = 7.5$ MeV. We use a finite-range residual interaction, the Surface Gaussian Interaction (SGI) [\[6\]](#page-1-5):

$$
V_{J,T}(\mathbf{r}_1,\mathbf{r}_2)=V_0(J,T)\cdot e^{-\left(\frac{\mathbf{r}_1-\mathbf{r}_2}{\mu}\right)^2}\cdot \delta(|\mathbf{r}_1|+|\mathbf{r}_2|-2\cdot R_0)
$$

together with the WS potential with the "5He" parameter set. The "5He" WS basis is undesirable when applied to the neutron-rich isotopes of He. Therefore, we use an optimized s.p. basis, i.e., the Hartree-Fock basis extended to unbound states. Such an optimal Berggren basis which

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Fig. 1. Energy splitting of the lowest $3/2^-$ and $1/2^-$ states of 5 He and ⁷He as a function of the spin-orbit strength $V_{\rm so}$. The solid lines with filled symbols show the truncated GSM results (see ref. [\[16\]](#page-1-15) for details of calculations). The $0p_{3/2}$ state of the GHF basis is unbound in the $GSM₁$ branch and bound in the GSM² branch. The solid line with empty squares represents the SM results for ⁷He obtained in the HO-SM approximation. The solid line with empty circles shows the splitting of the ⁵He $3/2^-$ and $1/2^-$ states, *i.e.*, the $0p_{1/2}$ and $0p_{3/2}$ eigenstates of the WS potential.

is generated in the Gamow-Hartree-Fock (GHF) approach allows for a more precise description of heavier p -shell nuclei [\[16\]](#page-1-15).

In the chain of helium isotopes, which are described assuming an inert ⁴He core, there are only $T = 1$ twobody matrix elements. Consequently, only $(J = 0, T = 1)$ and $(J = 2, T = 1)$ couplings come into play. We have adjusted $V_0(J = 0, T = 1)$ to reproduce the experimental ground state $(g.s.)$ energy of 6 He relative to the g.s. of ⁴He, whereas $V_0(J = 2, T = 1)$ has been fitted to all g.s. energies from 7 He to 10 He. The adopted values are: $V_0(J = 0, T = 1) = -403 \text{ MeV} \cdot \text{fm}^3$ and $V_0(J = 2, T = 1) = -315 \,\text{MeV} \cdot \text{fm}^3$. The experimental g.s. binding energies relative to the ⁴He core are reproduced fairly well with this interaction. For instance, the g.s. of ⁶He and ⁸He are bound, whereas g.s. of ⁵He and ⁷He are unbound. Moreover, the so-called helium anomaly [\[17\]](#page-1-16), i.e., the presence of the higher one- and two-neutron emission thresholds in ⁸He than in ⁶He, is well reproduced.

2.1 Spin-orbit effects: example of 7 He

The coupling to the particle continuum may be singular for $\ell = 0, 1$ orbits. In p-shell nuclei, this effect may lead to strong modifications of spin-orbit effects. In fig. [1](#page-1-17) we show the energy splitting between the lowest $3/2^-$ and $1/2^$ states of ⁷He as a function of $V_{\rm so}$ (all details of calculations follow ref. $[16]$. To compare the GSM results with standard SM, we calculate matrix elements of the SGI interaction in the harmonic-oscillator basis. The s.p. energies in such "equivalent SM calculations" (HO-SM approximation) are given by real parts of $0p_{1/2}$ and $0p_{3/2}$ eigenvalues of the WS potential generating the GSM basis. One can see that the $3/2^-$ – $1/2^-$ splitting is enhanced by the coupling to the non-resonant continuum states. The discontinuity seen between the two GSM results (GSM_1) and GSM_2) is an artifact of truncations used, especially neglect of 3p-3h excitations to the non-resonant continuum [\[16\]](#page-1-15).

3 Conclusions

The coupling to the non-resonant continuum increases when approaching the particle emission thresholds. The novel feature, absent in the standard SM, is a strong influence of S-matrix poles on the weakly bound/unbound many-body states. For the low- ℓ orbits $(\ell = 0, 1)$, the continuum coupling may induce the non-perturbative rearrangement of the wave function. We have demonstrated that the continuum coupling may be seen in the enhancement of spin-orbit effects for nuclei close to the driplines. A similar mechanism may also influence other observables such as the spectroscopic factors, pair-transfer amplitudes, nuclear collectivity, and properties of nuclear excitations.

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